Number System- Keynotes

Number Systems

When we consider a number in a decimal system we can divide it into units, tens, hundreds, one tenth, one hundredth etc. For example the numeral 572.65 can be written as $(5*10^{2}) + (7*10^{1}) + (2*10^{0}) + (6*10^{-1}) + (5*10^{-2})$.

We say that “10” is the base of the number system.

Base

The number which decides the place value of a symbol or a digit in a number. Alternatively, it is the number of distinct symbols that are used in that system. The base should be a positive integer other than 1. If N is any integer, r is the base of the system and a_0, a_1, a_2… a_n be the digits required to present N, then

$N= a_n r^n + a_{n-1} r^{n-1} + \ldots\ldots + a_1 r + a_0$, where $0 \le a_i \le r-1$

Eg: (i) $(143)_5 = 1*5^2 + 4*5^1 + 3*5^0 = 48$

(ii) $(1101)_2 = 1*2^3 + 1*2^2 + 0*2^1 + 1*2^0 = 13$.

Note: The subscript indicates the base. In the above examples 5 and 2 are bases. We can also represent fractions in other bases. For example $(0.572)_8 = 5*1/8 + 7*1/8^2 + 2*1/8^3$.

The following table lists some number systems along with their base and symbols.

Number System	Base	Symbols
Binary	2	0,1
Septenuary	7	0,1,2,3,4,5,6
Octal	8	0,1,2,3,4,5,6,7
Decimal	10	0,1,2,3,4,5,6,7,8,9
Duo-decimal	12	0,1,2,3,4,5,6,7,8,9,A,B
Hexa decimal	16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

A=10, B=11, C=12, D=13, E=14, F=15, some books denote ten as “E” and eleven as “e”.

The conversion of a number from one base to the other and the arithmetic operations involving bases other than 10 are discussed in worked out examples.

We need to remember the elementary rules while adding binary numbers.

0+0=0
0+1=1
1+0=1
1+1=10
1+1+1=11

1.Convert $(216.42)_8$ into base 10.

Sol. (216.45)8 = $2*8^2 + 1*8^1 + 6*8^0 + 4*8^{-1} + 2*8^{-2}$

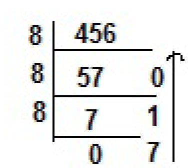
= 128+8+6+ ½ + 1/32 = (142.53125)10

2.Convert (1101.11)2 into base 10.

Sol. (1101.11)2 = 1*23 + 1*22 + 0*21 + 1*20 + 1*2-1 + 1*2-2

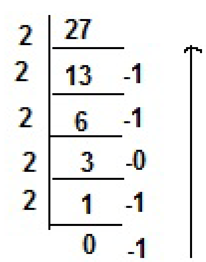
= 8+4+1+ ½ + ¼ = (13.75)10

3.Convert $(456)_{10}$ into base 8.



Thus $(456)_{10} = (710)_8$

4.Convert (27)10 into base 2.



Thus (27)10 =(11011)2

5.Find (1100101)2 + (110)2

		1				——— carry
1	0	0	1	0	1	
0	0	0	1	1	0 (Intoduce leading zeros)	
1	0	1	0	1	1	

6.Find (101110)2 + (111011)2

	1	1	1	1		——	(Carry)
	1	0	1	1	1	0	
	1	1	1	0	1	1	
1	1	0	1	0	0	1	

Exercise Questions

1. The binary equivalent of the decimal number 125 is

a.1100100 b.1111101 c.1101100 d.1111111

2. The hexa decimal equivalent of the decimal number 128 is

a.128 b.175 c.80 d.81

3. The decimal number 1356 expressed in octal system equals

a.2514 b.125 c.353 d.235

4. The decimal conversion of the binary number $(1111)_2$ is…..

a.31 b.15 c.13 d.14

5. The sum of $(101101)_2$ and $(111011)_2$ is

a.1010110 b.1101000 c.1000110 d.1110010

6. The square root of $(2011)_5$ is

a.$(21)_5$ b.$(31)_5$ c.$(121)_5$ d.$(41)_5$

7. The sum of $(6E)_{16}$ and $(3B)_{12}$ is

a.$(157)_{10}$ b.$(137)_{11}$ c.$(166)_8$ d.$(192)_7$

8. The decimal equivalent of hexa-decimal number $(ABC)_{16}$

a.2847 b.2748 c.7428 d.1478

9. The decimal fraction 0.75 in the binary system equals

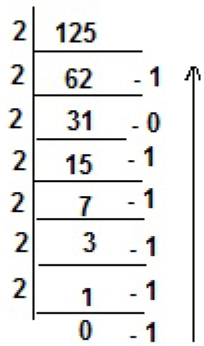
a.0.11 b.0.00 c.0.10 d.0.111

10. The octal equivalent to the binary $(11010)_2$ is

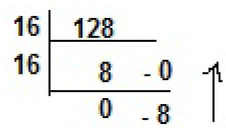
a.26 b.32 c.28 d.30

Answer & Exaplanations

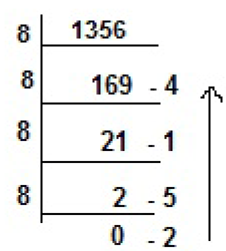
1. Ans: (b).



2.Ans: (c).



3.Ans: (a).



4.Ans: (b)

$(1111)_2 = 1*2^3 + 1*2^2 + 1*2^1 + 1*2^0 = 8+4+2+1 = 15$

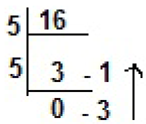
5.Ans: (b).

	1	1	1	1	1		(carry)
	1	0	1	1	0	1	
	1	1	1	0	1	1	
1	1	0	1	0	0	0	

6.Ans: (b).

$(2011)_5 = 2*5^3 + 0*5^2 + 1*5^1 + 1*5^0 = 250+5+1 = (256)_{10}$

The square root of $(256)_{10} = (16)_{10} = (31)_5$



7.Ans: (a)

$(6E)_{16} = 6*16^1 + 14*16^0 = 96 + 14 = (110)_{10}$

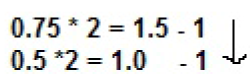
$(3B)_{12} = 3*12^1 + 11*12^0 = 36 + 11 = (47)_{10}$

$(110)_{10} + (47)_{10} = (157)_{10}$

8. Ans: (b)

$(ABC)_{16} = 10*16^2 + 11*16^1 + 12*16^0 = 2560+176+12 = 2748.$

9.Ans: (a)



10.$(11010)_2 = 1*2^4 + 1*2^3 + 0*2^2 + 1*2^1 + 0*2^0 = 16+8+2 = (26)_{10}$